Hertzsprung-Russell Diagram

Relativity and Astrophysics
Lecture 14
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Outline

- Magnitudes
- Hertzsprung-Russell Diagram
  - Main-sequence, giant, supergiants, and white dwarfs
- Summary of M-S stellar properties
  - Mass, size, temperature, lifetime

Magnitudes

- We would like a way of specifying the relative brightness of stars
- Hipparchus
  - Devised a the magnitude system 2100 years ago to classify stars according to their apparent brightness.
  - He labeled 1080 stars as class 0, 1,.. 6.
  - 0 was the brightest, 1 the next brightest, etc.
- The magnitude scale is logarithmic.

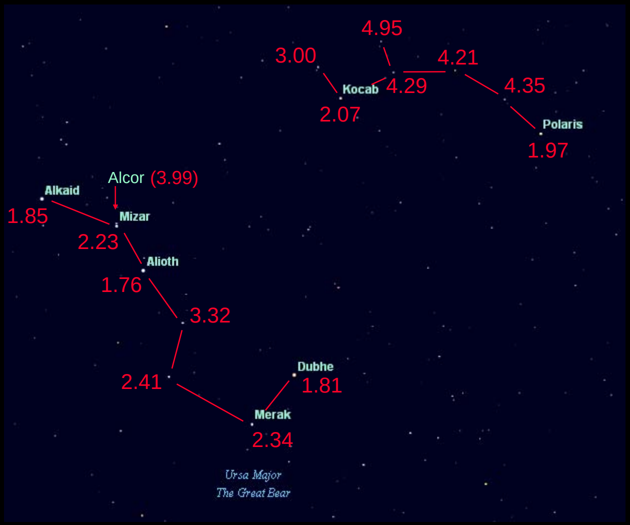
- An increase in magnitude by 2.5 means an object is a factor of 10 dimmer, e.g.
  - a 0 mag star is 10 times brighter than a 2.5 mag star.
  - a 0 mag star is 100 times brighter than a 5 mag star.

Example magnitudes

Star or Planet	m_v	Designations/Comment
Sun	-26.8	
Sirius	-1.47	α CMa
Canopus	-0.72	α Car
Arcturus	-0.06	α Boo
Vega	0.03	α Lyr
Betegeuse	0.45	β Ori
Altair	0.77	α Aqu
Deneb	1.26	α Cyg
Venus	-4.6 to -3.8	range
Mars, Jupiter	-2.9	max
Saturn	-0.4	max

- A dark adapted person with good eyesight can see to ~ 6th magnitude.
- Hubble Space Telescope can observed objects fainter than 30 mag.
  - 4×10^9 times fainter than the eye!

Big and Little Dippers



You can barely see 4th magnitude stars from north campus.

Visual apparent magnitudes in red.

Fluxes and Magnitudes

- Flux is the power per unit area received from an object, e.g. $f_{\text{sun}} = 1\ \text{kW/m}^2$
- If two stars, A and B, have fluxes, f_A and f_B, their magnitudes are related by

$$m_A - m_B = -2.5\log(f_A / f_B)$$

  - Thus if $f_B / f_A = 10$, then $m_A - m_B = 2.5$
- We can also write the inverse relation

$$\frac{f_B}{f_A} = 10^{\frac{m_A - m_B}{2.5}} \quad \text{or} \quad \frac{f_B}{f_A} = 2.512^{(m_A - m_B)}$$

  - So that if $m_A = 5$ and $m_B = 0$, $f_B / f_A = 100$.

Each magnitude is a factor of 2.512 and 2.5 mag. is a factor 10.

Absolute & Bolometric Magnitudes

- m_v – apparent magnitude
  - How bright a star appears in the sky.
- M_v – absolute magnitude
  - Brightness if the star were at 10 pc
  - This is an intrinsic property of the star!
- M – absolute bolometric magnitude
  - Brightness at ALL wavelengths (and 10 pc).

- To get M_v or M we must know the distance to the star.
- Example:
  - Suppose a star has m_v = 7.0 and is located 100 pc away.
  - It is 10 times the standard distance, thus, it would be 100 times brighter to us at the standard distance.
  - Or 5 magnitudes brighter => $M_v = 2.0$

Example Absolute Magnitudes

Object	m_V	M_V
Sun:	-26.8	4.77
Full Moon:	-12.6	(32)
Sirius:	-1.47	1.4
Canopus:	-0.72	-3.1
Arcturus:	-0.06	-0.3
Deneb:	1.26	-7.2

The Distance Modulus Equation

- The relation between m_v and M_v is written in equation form as:

$$m_\mathrm{v} - M_\mathrm{v} = -5 + 5\log_{10}(d) \qquad (d \text{ in pc})$$

- $m_\mathrm{v} - M_\mathrm{v}$ is called the distance modulus.
- Examples:
  - Deneb: m_v = 1.26 and is 490 pc away.

$$m_\mathrm{v} - M_\mathrm{v} = -5 + 5\log_{10}(d)$$
$$1.26 - M_\mathrm{v} = -5 + 5\log_{10}(490) = -8.5$$
$$\Rightarrow \quad M_\mathrm{v} = -7.2$$

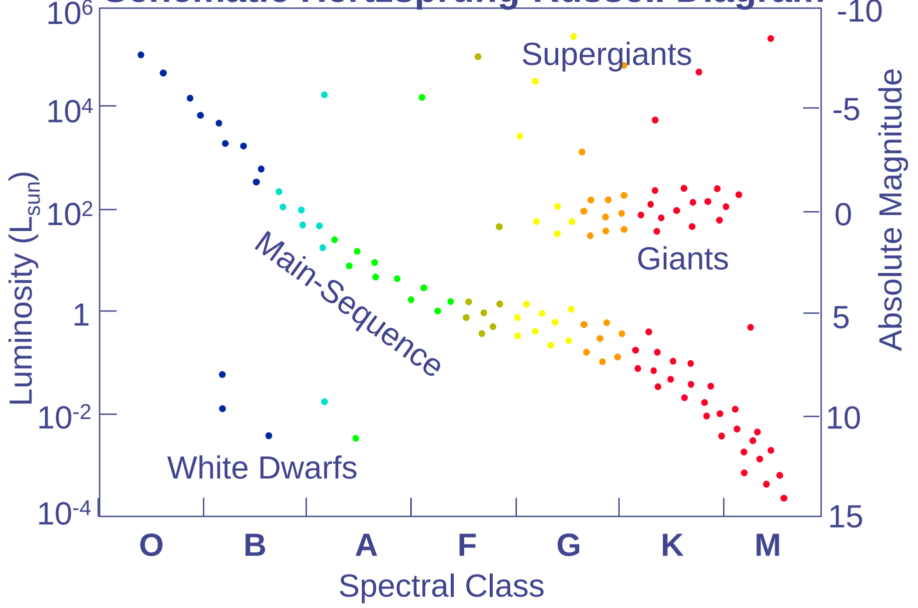
  - Sun: $m_\mathrm{v} = -26.8$, $d = 1$ AU

$$-26.8 - M_\mathrm{v} = -5 + 5\log_{10}(1/206265)$$
$$\Rightarrow \quad M_\mathrm{v} = 4.8$$

Luminosity vs. Color of Stars

- In 1911, Ejnar Hertzsprung investigated the relationship between luminosity and colors of stars in within clusters.
- In 1913, Henry Norris Russell did a similar study of nearby stars.
- Both found that the color (temperature, spectral type) was related to the luminosity.

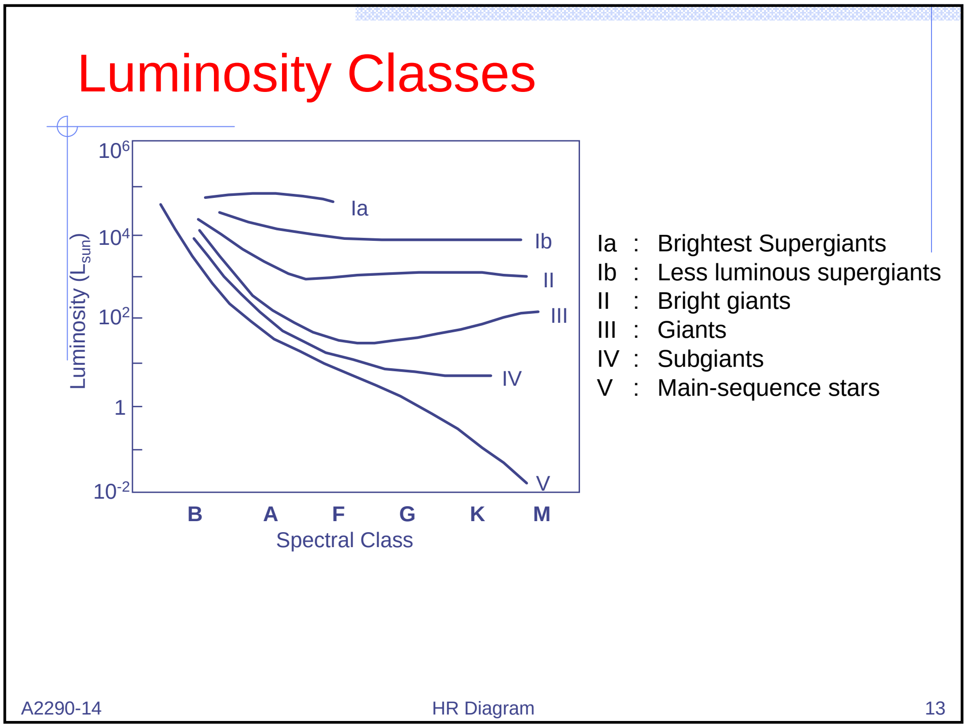
Schematic Hertzsprung-Russell Diagram



Notes on H-R Diagram

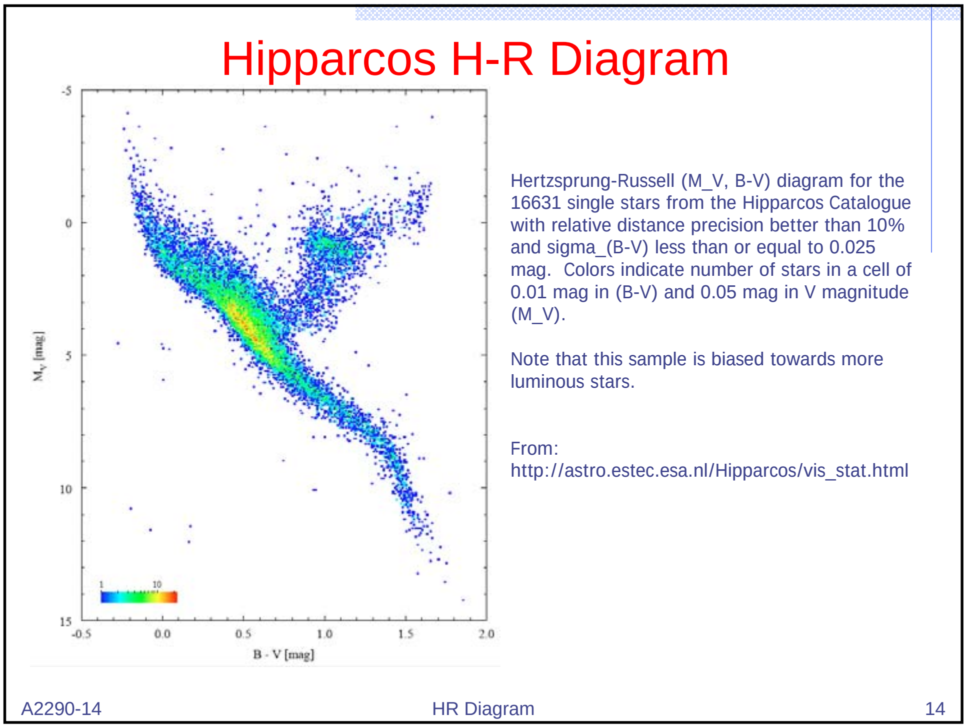
- There are different regions
  - main sequence, giant, supergiant, etc.
- Most stars lie along the main-sequence.
- For a given spectral class (e.g. K), there can be more than one luminosity.
  - i.e. main-sequence, giant or supergiant
- On the main sequence, there are many more K and M stars than O and B stars.

- Observational Effects
  - An H-R diagram of the brightest stars will preferentially show luminous star because we can see them farther away.
  - An H-R diagram of the nearest stars show many M type stars because M stars are very numerous.



Luminosity Classes

Ia	:	Brightest Supergiants
Ib	:	Less luminous supergiants
II	:	Bright giants
III	:	Giants
IV	:	Subgiants
V	:	Main-sequence stars

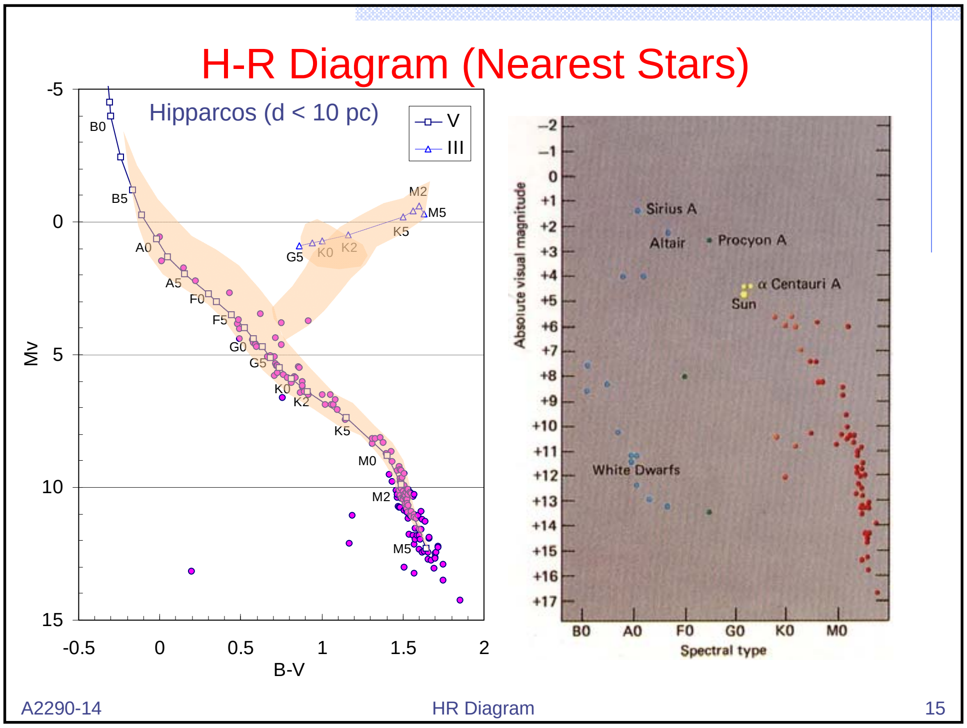


Hipparcos H-R Diagram

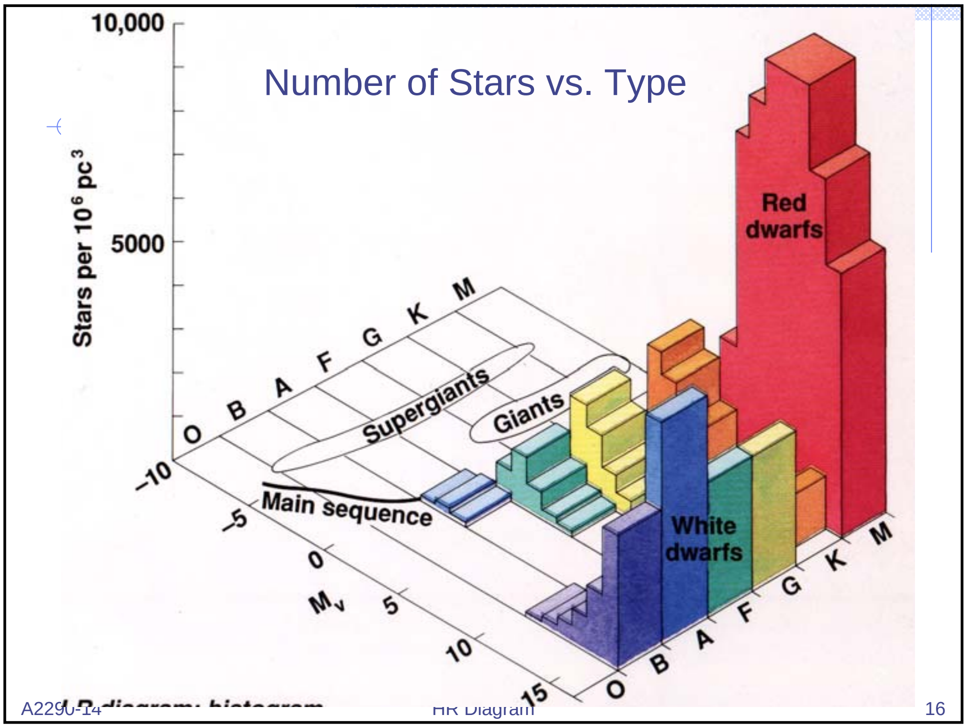
Hertzsprung-Russell (M_V, B-V) diagram for the 16631 single stars from the Hipparcos Catalogue with relative distance precision better than 10% and sigma_(B-V) less than or equal to 0.025 mag. Colors indicate number of stars in a cell of 0.01 mag in (B-V) and 0.05 mag in V magnitude (M_V).

Note that this sample is biased towards more luminous stars.

From:
http://astro.estec.esa.nl/Hipparcos/vis_stat.html



H-R Diagram (Nearest Stars)



Number of Stars vs. Type

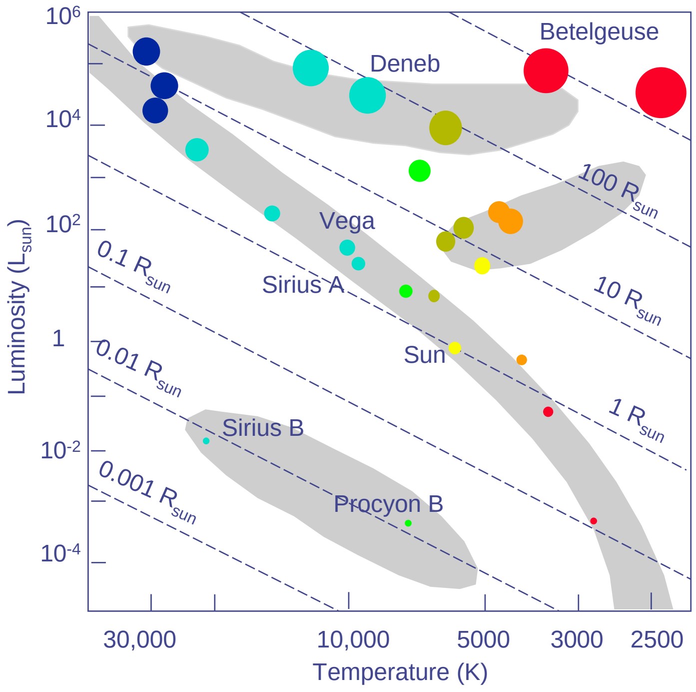
How big are supergiants?

- Using the expression relating luminosity, temperature and size we can compare a supergiant with the Sun.
- Betelgeuse: M2 Iab (supergiant)
  - $L \sim 40000\, L_{sun}$, $T \sim 3500$ K
- Sun: G2 V (main-sequence)
  - $T \sim 5800$ K

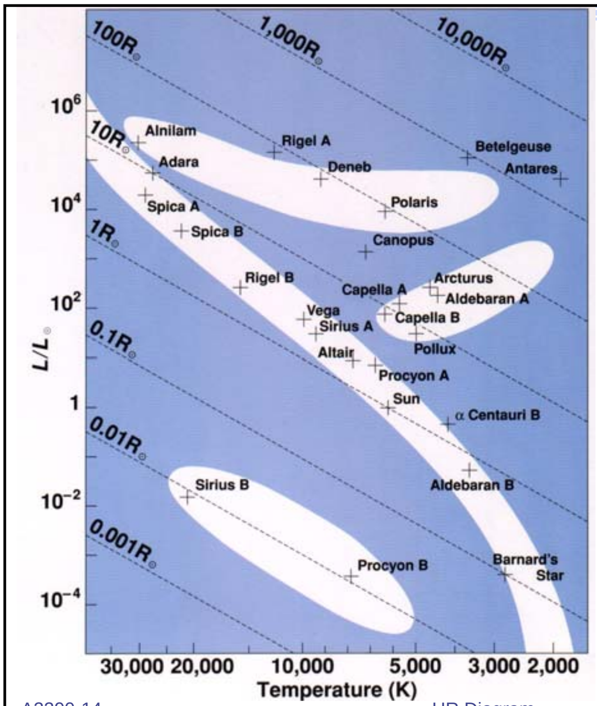
$$L = 4\pi R^2 \sigma T^4 \quad \Rightarrow \quad \frac{L_{\text{bet}}}{L_{\text{sun}}} = \frac{R_{\text{bet}}^2 T_{\text{bet}}^4}{R_{\text{sun}}^2 T_{\text{sun}}^4}$$

$$\Rightarrow \quad \frac{40{,}000 L_{\text{sun}}}{L_{\text{sun}}} = \left[\frac{R_{\text{bet}}}{R_{\text{sun}}}\right]^2 \left[\frac{3500}{5800}\right]^4$$

$$\Rightarrow \quad \boxed{R_{\text{bet}} \sim 550 R_{\text{sun}}}$$



H-R Diagram



Sun's radius is about 1/200 of an AU. So largest stars would extend beyond the orbit of the earth!!

Spectroscopic Parallax

- From a star's spectrum, we can determine its spectral and luminosity class.
- Given the star's apparent brightness (observed flux), we can then estimate its distance.
- This distance determination technique is called spectroscopic parallax

Example:

- Observe a G2 Ia star (supergiant) with
  - apparent magnitude $m_v = 10$.
- The absolute magnitude (from the H-R diagram) is $M_v = -5$.

but $m_v - M_v = -5 + 5\log_{10}(d)$

=> $\log_{10}(d) = 20/5 = 4$

=> $d = 10{,}000$ pc

Stellar Masses

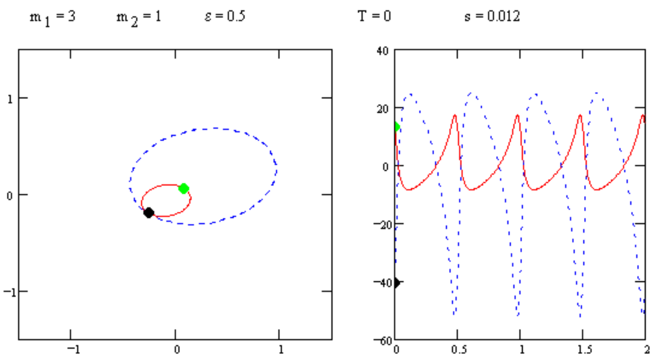
- We know many properties of stars now:
  - Temperature, radius, luminosity, surface composition
- But the most important determining characteristic of a star is its mass.

- How do we “weigh” a star?
- Binary stars (75% of all stars are “binary stars”)
  - pairs of stars that orbit each other
  - used to determine masses of stars

Type of Binaries

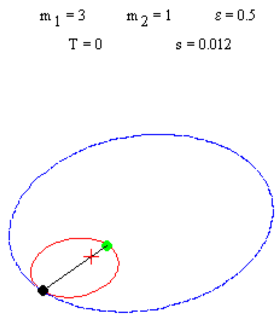
- Visual Binary
  - Stars are separated in a telescope
- Spectroscopic Binary
  - See two sets of spectral lines Doppler shifted due to orbital motion
- Eclipsing Binary (rare)
  - Stars cross in front of one another

Spectroscopic Binary Simulation



The Doppler shift shows the velocity changing periodically. The system is probably not a “visual” binary and you may only be able to detect one star.

Binary simulation



Cross indicates the “center of mass” of the system.
The stars orbit about this point.

Masses of Binary Stars

- Newton’s laws allow us to determine the total mass in a binary system.
- For star of mass M_A and M_B (in solar masses), the total mass is related to the period, P, in years and the average distance between the stars, a (in AU).

$$M_A + M_B = \frac{a^3}{P^2}$$

- Example:
  - If a visual binary has a period of 32 years and an average separation of 16 AU then

$$M_A + M_B = \frac{16^3}{32^2} = \frac{16 \times 16 \times 16}{32 \times 32} = \frac{16}{4} = 4\,M_{sun}$$

- Now with stellar masses in hand we can compile table with properties of stars

Summary of Main-Sequence Stellar Properties

Class	Mass (M_{sun})	L (L_{sun})	Temp. (K)	Radius (R_{sun})	Lifetime (10^6 yrs)
O5	40	400,000	40,000	13	1
B0	15	13,000	28,000	4.9	12
A0	3.5	80	10,000	3.0	440
F0	1.7	6.4	7,500	1.5	2,700
G0	1.1	1.4	6,000	1.1	7,900
K0	0.8	0.46	5,000	0.9	17,000
M0	0.5	0.08	3,500	0.8	57,000

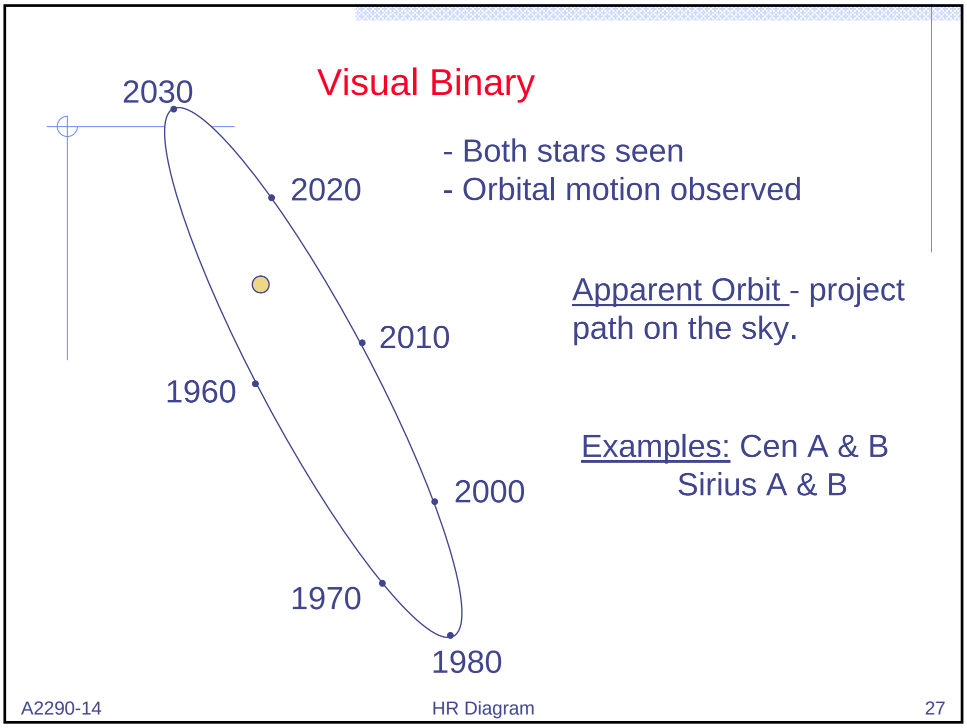
- The luminosity of stars on the main-sequence varies approximately as $L \propto M^{3.5}$ with mass.
- Since the fuel in stars is proportional to the mass, M, the lifetime of a star is roughly

$$t_{life} = \frac{\text{fuel}}{\text{burn rate}} = \frac{M}{M^{3.5}} = M^{-2.5}$$

Where M is in solar masses and t_{life} is in solar lifetimes (~ 10^{10} yrs).

Extra Slides

- Binary stars
- Deriving Kepler's Harmonic Law



Visual Binary

- Both stars seen
- Orbital motion observed

Apparent Orbit - project path on the sky.

Examples: Cen A & B
Sirius A & B

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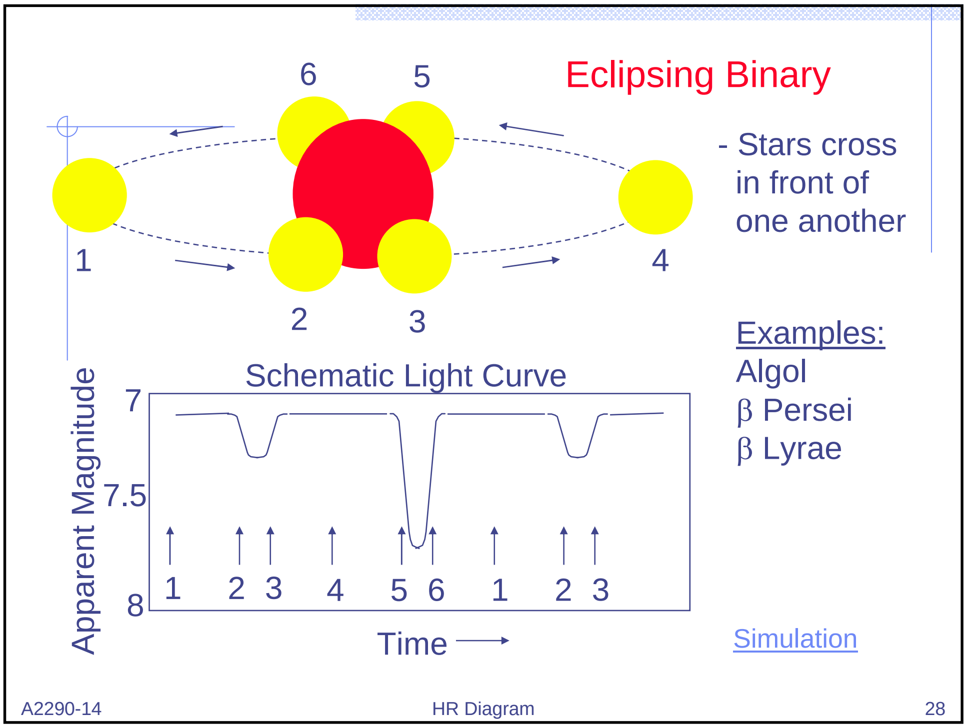

Eclipsing Binary

- Stars cross in front of one another

Examples:
Algol
β Persei
β Lyrae

Simulation

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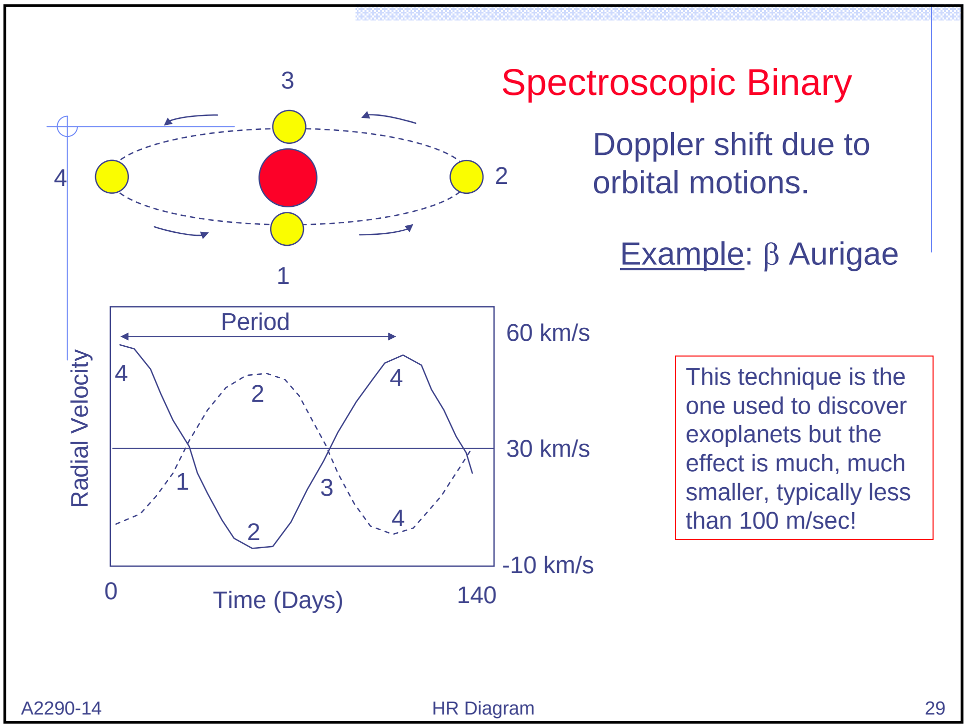


Spectroscopic Binary

Doppler shift due to orbital motions.

Example: β Aurigae

This technique is the one used to discover exoplanets but the effect is much, much smaller, typically less than 100 m/sec!

Bonus: Deriving Harmonic Law

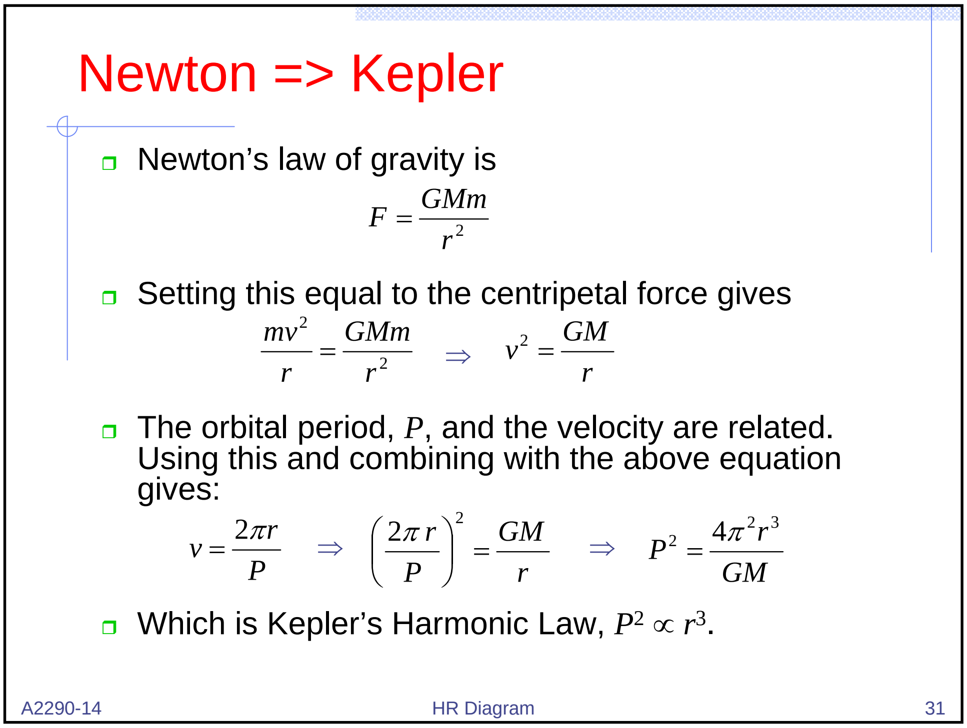
- The “Harmonic Law” of Kepler related period to distance in an orbit.
- Newton’s triumph was deriving this relationship
  - What follows is simplified derivation based on circular (rather than elliptical) orbits and $M >> m$.
- Start with Newton’s equation for acceleration by a force

$$F = ma$$

  - that is, force is mass times acceleration
- For a circular orbit, the (centripetal) acceleration is given by:

$$a = \frac{v^2}{r} \quad \Rightarrow \quad F = m\frac{v^2}{r}$$

- Now we use Newton’s law of gravity



Newton => Kepler

- Newton’s law of gravity is

$$F = \frac{GMm}{r^2}$$

- Setting this equal to the centripetal force gives

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \quad \Rightarrow \quad v^2 = \frac{GM}{r}$$

- The orbital period, P, and the velocity are related. Using this and combining with the above equation gives:

$$v = \frac{2\pi r}{P} \quad \Rightarrow \quad \left(\frac{2\pi r}{P}\right)^2 = \frac{GM}{r} \quad \Rightarrow \quad P^2 = \frac{4\pi^2 r^3}{GM}$$

- Which is Kepler’s Harmonic Law, $P^2 \propto r^3$.